Thermal evolution of the chiral condensate in SU(2) and SU(3) Chiral Perturbation Theory.

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The temperature evolution of the chiral condensates in a gas made of pions, kaons and etas is studied within the framework of SU(2) and SU(3) Chiral Perturbation Theory. We describe the temperature dependence of the quark condensates by using the meson meson scattering phase shifts in a second order virial expansion. We find that the SU(3) formalism yields an extrapolated melting temperature for the non-strange condensates which is lower by about 20-30 MeV than within SU(2). In addition our results show that the strange condensate melting is slower than that of the non-strange, due to the different strange and non-strange quark masses.

1. Introduction

We review here our recent study of the thermal evolution of the chiral strange and non-strange condensates [1]. Our motivation is the study of the the phase diagram of QCD at zero baryonic densities, which is a state that is expected to occur in the central rapidity regions of Relativistic Heavy Ion Collisions. Our purpose is to obtain a model independent description of the condensate and its thermal evolution. To that aim we will turn to the virial expansion and to Chiral Perturbation Theory.

The virial expansion is a simple and successful approach to describe many thermodynamic features of dilute gases made of interacting pions [2] and other hadrons [3]. Let us note that for most thermodynamic properties it is enough to know the low energy scattering phase shifts of the particles, which, in principle, could be taken from experiment, avoiding any model dependence. However, we are interested in the chiral symmetry restoration, whose order parameters are the quark condensates, which are defined as derivatives of the pressure with respect to the quark masses. These derivatives cannot be measured experimentally and therefore a model independent theoretical description of the scattering amplitudes mass dependence is required. That is why we need the second ingredient of our approach: Chiral Perturbation Theory (ChPT) [4–6], which provides a model-independent and systematic description of low energy hadronic interactions. ChPT follows from the identification of the pions, kaons and the eta as the Goldstone Bosons of the QCD spontaneous chiral symmetry breaking (pseudo Goldstone bosons indeed, due to the small light-quark masses). Then, the ChPT Lagrangian is built as the most general derivative and mass expansion, over $4\pi F \simeq 1.2\,\mathrm{GeV}$, (the symmetry breaking scale) respecting the symmetry constraints. At one loop any calculation can be renormalized in terms of just an small set of parameters, $L_k(\mu)$, $H_k(\mu)$, (μ being the renormalization scale) which can be determined from a few experiments and used for further predictions.

In particular, within SU(2) ChPT the leading coefficients of the low temperature expansion for the pressure and the non-strange quark condensate have been calculated for a hadron gas whose only interacting hadrons were the pions [7]. Adding other more massive particles (like kaons, etas, rhos, protons, etc...) as *free states*, it was possible to obtain an estimate, $T \simeq 190-200\,\mathrm{MeV}$, of the phase transition critical temperature. It was also shown that the perturbative calculation of the pressure was analogous to the second order virial expansion when the interacting part of the second virial coefficient is obtained from the one loop $\pi\pi$ ChPT scattering lengths. Other works have studied the applicability of the virial expansion with a pionic chemical potential [8], or the critical temperature in generalized scenarios [9].

The interest of extending the previously commented work to SU(3) is that the SU(2) approach is limited by the absence of other interacting particles, like kaons or etas, whose densities at $T \simeq 150$ MeV are significant [7]. In addition, let us recall that the chiral phase transition can be different for the SU(2) and SU(3) cases, and that several QCD inequalities and lattice results suggest an stronger chiral condensate temperature suppression with an increasing number of light flavors [10]. Intuitively this is due to the fact that the existence of other states (strange quarks, or strange mesons) makes it easier to create entropy, that is, disorder, and therefore it is easier to melt the ordered state, that is, the condensates. This effect has been observed in lattice calculations [11], but only in the chiral limit, and it seems to lower the chiral critical temperature down by roughly 20 MeV. Note, however, that none of these results have been obtained from the hadronic phase and realistic masses, which is what we have achieved with our approach.

In addition, within SU(2) it is not possible to study the $\langle \bar{s}s \rangle$ condensate. This could be interesting because, in the chiral phase diagram, quark masses play the same role as magnetic fields in ferromagnets: Intuitively, we need a higher temperature to disorder a ferromagnet when there is a magnetic field aligned along the direction of the magnetization. Analogously, it has been found that the SU(2) chiral condensate melts at a lower temperature in the massless limit [7]. In particular, we have obtained the temperature dependence of $\langle \bar{s}s \rangle$ in the hadronic phase. Since the strange quark mass m_s is much larger than m_u, m_d, we have indeed found a sizable "ferromagnetic" effect which translates into a slower thermal $\langle \bar{s}s \rangle$ evolution. This effect had already been studied in the large N_c limit with the use of QCD-motivated effective Lagrangian [12].

Thus, the second order relativistic virial expansion of the pressure for an inhomogeneous gas made of three species: $i = \pi, K, \eta$ is [13,2]:

$$\beta P = \sum_i B_i(T)\xi_i + \sum_i \left(B_{ii}\xi_i^2 + \frac{1}{2}\sum_{j\neq i} B_{ij}\xi_i\xi_j \right) ..., \quad (1)$$

where $\beta = 1/T$ and $\xi_i = \exp(-\beta m_i)$. Expanding up to the second order in ξ_i means that we consider only binary interactions. For a free boson gas, $B_{ij}^{(0)} = 0$ for $i \neq j$, whereas:

$$B_i^{(0)} = \frac{g_i}{2\pi^2}\int_0^\infty dp\, p^2 e^{-\beta(\sqrt{p^2+m_i^2}-m_i)}, \quad (2)$$

$$B_{ii}^{(0)} = \frac{g_i}{4\pi^2}\int_0^\infty dp\, p^2 e^{-2\beta(\sqrt{p^2+m_i^2}-m_i)}. \quad (3)$$

The degeneracy is $g_i = 3, 4, 1$ for π, K, η, respectively. The interactions appear through [13,7]:

$$B_{ij}^{(int)} = \frac{\xi_i^{-1}\xi_j^{-1}}{2\,\pi^3}\int_{m_i+m_j}^\infty dE\, E^2 K_1(E/T)\Delta^{ij}(E),$$

where K_1 is the first modified Bessel function and:

$$\Delta^{ij} = \sum_{I,J,S}(2I+1)(2J+1)\delta_{I,J,S}^{ij}(E), \quad (4)$$

$\delta_{I,J,S}^{ij}$ being the $ij \to ij$ phase shifts (chosen so that $\delta = 0$ at threshold) of the elastic scattering of a state ij with quantum numbers I, J, S (J being the total angular momentum and S the strangeness).

The virial expansion breaks around $T \simeq 200-250$ MeV [8]. Fortunately, the physics we are interested in occurs already at $T < 250$ MeV, and $\xi_\pi > \xi_K \simeq \xi_\eta$, and it is enough to consider $ij = \pi\pi, \pi K$ and $\pi\eta$ in B_{ij}.

At this point we recall that the quark mass appears in the QCD Lagrangian, and therefore in the partition function, as $m_{q_\alpha}\bar{q}_\alpha q_\alpha$, where $q_\alpha = u, d, s$. Thus, in order to obtain the condensate we simply have to differentiate the partition function with respect to m_{q_α}. At finite temperature, the partition function is substituted by the free energy density z, so that [7]

$$\langle \bar{q}_\alpha q_\alpha \rangle = \frac{\partial z}{\partial m_{q_\alpha}} = \langle 0|\bar{q}_\alpha q_\alpha|0\rangle - \frac{\partial P}{\partial m_{q_\alpha}}. \quad (5)$$

Note that we have separated the $T = 0$ part from the temperature dependent part, which is nothing but the pressure $P=\epsilon_0 - z$, ϵ_0 being the vacuum energy density. Of course, at $T=0$ we recover $\langle\bar{q}_\alpha q_\alpha\rangle = \langle 0|\bar{q}_\alpha q_\alpha|0\rangle = \partial\epsilon_0/\partial m_{q_\alpha}$.

Once more, we emphasize that in contrast with most thermodynamic quantities, for the chiral condensate it is not enough to know the $\delta(E)$, but *we also need their dependence with the quark masses* as well as a value for the $T=0$ vacuum expectation value. We will obtain this information from ChPT. Nevertheless, ChPT does not deal with quarks, and thus we first have to rewrite the condensate, eq.(5), in terms of meson masses:

$$\langle\bar{q}_\alpha q_\alpha\rangle = \langle 0|\bar{q}_\alpha q_\alpha|0\rangle\left(1+\sum_i \frac{c_i^{\bar{q}_\alpha q_\alpha}}{2m_i F^2}\frac{\partial P}{\partial m_i}\right) \quad (6)$$

where, as before, $i=\pi, K, \eta$, and we have defined:

$$c_i^{\bar{q}_\alpha q_\alpha} = -F^2 \frac{\partial m_i^2}{\partial m_{q_\alpha}}\langle 0|\bar{q}_\alpha q_\alpha|0\rangle^{-1}. \quad (7)$$

In the isospin limit, when $m_u = m_d$, the u and d condensates are equal, and we define $\langle 0|\bar{q}q|0\rangle \equiv \langle 0|\bar{u}u + \bar{d}d|0\rangle$. It is tedious but straightforward to obtain the c parameters above, whose explicit expressions can be found in [1]. The only relevant comment is that the c coefficients depend on the chiral parameters L_k, for $k=4...8$, and H_2.

In Table I, we show two L_k determinations from meson data, but it makes little difference to use other parameter sets. For H_2 we will use $H_2^r(M_\rho) = (-3.4\pm 1.1)10^{-3}$, obtained as explained in [14] but using a more recent estimation of $\langle 0|\bar{s}s|0\rangle/\langle 0|\bar{q}q|0\rangle = 0.75\pm 0.12$ [15]. For instance, the c parameters obtained when using the L_k in the first column of Table I are:

$$c_\pi^{\bar{q}q} = 0.9^{+0.2}_{-0.4}, c_K^{\bar{q}q} = 0.5^{+0.4}_{-0.7}, c_\eta^{\bar{q}q} = 0.4^{+0.5}_{-0.7},$$
$$c_\pi^{\bar{s}s} = -0.005^{+0.029}_{-0.037}, c_K^{\bar{s}s} = 1.3^{+0.4}_{-0.8}, c_\eta^{\bar{s}s} = 1.5^{+0.9}_{-1.6}.$$

The above $c_\pi^{\bar{q}q}$ is in good agreement with the SU(2) estimates: 0.85 and 0.90 ± 0.05 [7].

Finally, we have to introduce the meson-meson interactions in the second order virial coefficients through the phase shifts in eq.(4). These are nothing but the complex phases of the meson-meson amplitudes, once they are projected in partial waves of definite isospin and angular momentum. The one loop ChPT amplitudes have been given in [16,17]. As already commented within SU(2) ChPT it has been shown [7] that using just the amplitudes at threshold (scattering lengths) in the virial expansion is equivalent to expanding the partition function to third order in T/F or T/m_π. This approach yields a fairly good representation of the pion gas thermodynamics at low temperatures, $T \ll 150\,\text{MeV}$ [7]. Let us recall that ChPT provides a good low energy description ($E < 500\,\text{MeV}$) of the meson-meson amplitudes [16].

$\mu = M_\rho$	Refs.[5,21]	IAM [17]
L_1^r	0.4 ± 0.3	$0.561\pm 0.008\,(\pm 0.10)$
L_2^r	1.35 ± 0.3	$1.21\pm 0.001\,(\pm 0.10)$
L_3	-3.5 ± 1.1	$-2.79\pm 0.02\,(\pm 0.12)$
L_4^r	-0.3 ± 0.5	$-0.36\pm 0.02\,(\pm 0.17)$
L_5^r	1.4 ± 0.5	$1.4\pm 0.02\,(\pm 0.5)$
L_6^r	-0.2 ± 0.3	$0.07\pm 0.03\,(\pm 0.08)$
L_7	-0.4 ± 0.2	$-0.44\pm 0.003\,(\pm 0.15)$
L_8^r	0.9 ± 0.3	$0.78\pm 0.02\,(\pm 0.18)$

Table 1
Different sets of chiral parameters $\times 10^3$. In the first column L_1^r, L_2^r, L_3 are taken from [21] and the rest from [5] (L_4^r and L_6^r are estimated from the Zweig rule). The last column is the IAM fit [17] to meson-meson scattering up to 1.2 GeV.

The virial coefficients and $\partial P/\partial m_i$ have been calculated numerically. For simplicity, in the figures, we have represented the chiral condensate over its vacuum expectation value, $\langle\bar{q}_\alpha q_\alpha\rangle/\langle 0|\bar{q}_\alpha q_\alpha|0\rangle$, so that all of them are normalized to 1 at $T=0$. As a matter of fact, since there is always an small explicit chiral symmetry breaking due to the quark masses, the condensate should only vanish completely in the $T\to\infty$ limit (following with our analogy between the quark masses and the magnetic field, our ferromagnet above the Curie point becomes paramagnetic, but as if it was in the presence of a magnetic field, which still produces some magnetization.). Of course, with a virial expansion truncated at second order we cannot generate such an analytic behavior, and our curves become negative above some T, where the approach becomes clearly unreliable. Nevertheless, since the u and d masses

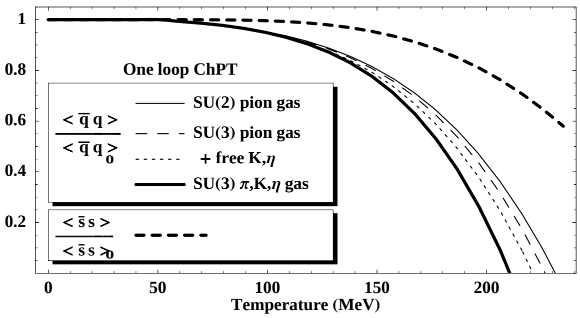

Figure 1. Condensate thermal evolution using one-loop ChPT amplitudes. Although *they should not really vanish* at a finite temperature, they are extrapolated down to zero only for reference.

are so small compared with the size of the non-strange condensate at T=0 (less than 3%), it is a fairly good approximation to speak about a melting temperature in that case. For the $\langle \bar{s}s \rangle$ condensate, we will only consider an extrapolated melting temperature to ease the comparison with the non-strange case, but that number does not have an actual physical meaning.

In Figure 1 we show the results of using the one-loop ChPT phase shifts obtained using the central values of the parameters in the first column of Table I. For illustration, we have separated the different contributions. In particular, the thin continuous line represents the interacting pion gas in SU(2), (see [6] for the conversion of SU(3) into SU(2) parameters), whereas the thin-dashed line is the corresponding one-loop SU(3) result also for a pion gas. We have checked that the tiny difference between them comes only from the pure $O(p^4)$ phase shift contribution, (the amplitude, once the parameters are translated, is the same for SU(2) and SU(3) only up to s/M_K^2 or s/M_η^2 terms coming from the expansion of kaon or eta loops, which, of course, are not present in SU(2) [6]). This amounts to a 4 MeV decrease of the pion gas extrapolated melting temperature: $T_m^{\langle \bar{q}q \rangle} = 231\,\mathrm{MeV}$ for SU(2) and $T_m^{\langle \bar{q}q \rangle} = 227\,\mathrm{MeV}$ for SU(3). Next, the thin-dotted line is obtained by adding free kaons and etas to the pion gas, which lowers the extrapolated melting temperature, by roughly 5-6 MeV down to 221 MeV.

Our SU(3) result for a π, K, η gas, where also the *kaons* and *etas* interact with each other and the pions, is the thick continuous line, where we see that the additional decrease, basically due to the $\pi K, \pi\eta$ interactions, amounts to 10-11 MeV. On a first glance, it may seem surprising that the $\pi K, \pi\eta$ interaction effect comes out comparable or larger than that of free kaons and etas, since it is thermally suppressed by another $\exp(-m_\pi/T)$ factor. Note, however, that the $\exp(-m_\pi/T)$ suppression only amounts to a factor of $6, 4, 2.5$ at T= 80,100,150 MeV, respectively. In contrast, the πK and $\pi\eta$ interactions depend strongly on m_π, which is much more sensitive to $\hat{m}$ than m_K or m_η, which carry the only $\hat{m}$ dependence of the free K and η terms. In particular, it is easy to see that $\partial m_\pi^2/\partial\hat{m} \simeq 2\,\partial m_K^2/\partial\hat{m}$ and $\partial m_\pi/\partial\hat{m} \simeq 6\,\partial m_K/\partial\hat{m}$, and this "temperature independent enhancement" competes with the thermal suppression, making the πK and $\pi\eta$ interaction effect comparable to the free one, already at $T = 80 - 100\,\mathrm{MeV}$ and even larger if $T > 140\,\mathrm{MeV}$.

Putting all the pieces together, the $T_m^{\langle \bar{q}q \rangle}$ decrease in SU(3) is roughly 20 MeV, in agreement with the chiral limit lattice results quoted in [11]: $T_c = 173\,\mathrm{MeV}$ for SU(2) and $T_c = 154\,\mathrm{MeV}$, for SU(3). The effect of using the real masses would be to increase both temperatures.

Furthermore, it can be noticed that $\langle \bar{s}s \rangle$ melts much slower than $\langle \bar{q}q \rangle$, since $m_s \gg \hat{m}$. Indeed, there is still 70% left of the $\langle \bar{s}s \rangle$ condensate at the $\langle \bar{q}q \rangle$ melting point.

Let us recall that both effects are already sizable at low temperatures $T \simeq 100\,\mathrm{MeV}$, where we can still trust pure ChPT. As commented above, only for illustration and to ease the comparison between curves and with previous works, we have extrapolated the condensates down to zero.

At this point we can also consider the uncertainties in the chiral parameters in Table I. By performing a Montecarlo gaussian sampling of the parameters within their errors, we can obtain a rather *conservative* estimate of our uncertainties (since the sampling is done assuming uncorrelated errors). We thus find:

$$T_m^{\langle \bar{q}q \rangle} = 211^{+19}_{-7}\,\mathrm{MeV},\ T_m^{\langle \bar{s}s \rangle} = 291^{+37}_{-35}\,\mathrm{MeV}$$

Note that the $\langle \bar{s}s \rangle$ extrapolation is beyond the reliability region of the second order virial expansion, is just illustrative, and really only vanishes in the $T \to \infty$ limit. Let us also remark that the extrapolated melting temperatures are strongly correlated, so that their difference has less uncertainty than simply adding in quadrature their respective errors. In particular, we find

$$T_m^{\langle \bar{q}q \rangle, SU(2)} - T_m^{\langle \bar{q}q \rangle, SU(3)} = 21^{+14}_{-7}\,\mathrm{MeV}$$
$$T_m^{\langle \bar{s}s \rangle} - T_m^{\langle \bar{q}q \rangle} = 80^{+25}_{-40}\,\mathrm{MeV}$$

We remark once more that the $\langle \bar{s}s \rangle$ melting temperature is, of course, only a crude extrapolation for illustrative purposes and to ease the comparison with the non-strange condensate evolution.

Furthermore, we have estimated the effect of other, more massive, hadrons. In the SU(2) case [7,8] these states also included the kaons and etas, although assuming a rather large uncertainty in $\partial M_i / \partial m_{q_\alpha}$. All in all this effect reduced $T_m^{\langle \bar{q}q \rangle}$ by approximately 10-20 MeV. In the SU(3) case we have indeed calculated explicitly the kaon and eta $\hat{m}$ dependence in the $c^{\bar{q_\alpha} q_\alpha}$ parameters, thus reducing considerably the above uncertainty. Since the kaons and eta are treated explicitly, the other massive hadrons are heavier than m_η, have a very low density, and their main contribution to the pressure comes from the first virial coefficient, i.e. the free gas. The only uncertainty is on $\partial M_h / \partial m_{q_\alpha}$, conservatively estimated to lie within the number of valence quarks N_{q_α} and $2N_{q_\alpha}$. Thus, we have found that their contribution decreases $T_m^{\langle \bar{q}q \rangle}$ by 7-12 MeV, respectively.

Finally, and in order to estimate the effects of higher energies we can extend ChPT up to $E \simeq 1.2\,\mathrm{GeV}$ by means of unitarization models [18,19,17]. These techniques resum the ChPT series respecting unitarity but also the low energy expansion, *including the mass dependency.* In particular, it has been shown that the coupled channel Inverse amplitude Method (IAM) provides a remarkable and accurate description of the complete meson-meson interactions below 1.2 GeV, generating dynamically six resonances from the one-loop ChPT expansion and a set of fitted L_k compatible with other previous determinations. This approach can be extended systematically to higher orders (for SU(2) case, see [20]).

Hence, in Figure 2 we show the results of using the one-loop coupled channel IAM fitted phase shifts. Its corresponding L_i parameters are given in the last column of Table I, with two errors, the first, very small, is purely statistical, and the second covers the uncertainty in the parameters depending on what systematic error is assumed for the experimental data. Let us remark that, although the L_k are highly correlated, the second, larger error, ignores completely these correlations, and should be considered as a very conservative range. The continuous line corresponds to the central values, and the dark shaded areas cover the one standard deviation uncertainty due to the small errors in the parameters. These areas have been obtained from a Montecarlo Gaussian sampling. The conservative ranges are covered by the light gray areas. We now find:

$$T_m^{\langle \bar{q}q \rangle} = 204^{+3}_{-1}\,\binom{13}{5}\,\mathrm{MeV}, T_m^{\langle \bar{s}s \rangle} = 304^{+39}_{-25}\,\binom{120}{65}\,\mathrm{MeV}$$

where the errors in parenthesis are the conservative errors which should be interpreted as uncertainty ranges better than as standard deviations. Note the excellent agreement with standard ChPT. The magnitude of the different contributions is roughly the same, although the πK and $\pi\eta$ interactions this time lower $T_m^{\langle \bar{q}q \rangle}$ by 17 MeV, and the free kaons and etas by roughly 10 MeV. Again other massive states apart from kaons and etas would lower $T_m^{\langle \bar{q}q \rangle}$ by 6-10 MeV, and $T_m^{\langle \bar{s}s \rangle}$ by 22-32 MeV. In addition, we find

$$\Delta T_m = T_m^{\langle \bar{q}q \rangle SU(2)} - T_m^{\langle \bar{q}q \rangle SU(3)} = 31.50^{+1.20}_{-0.03}\,\binom{9}{8}\,\mathrm{MeV}$$
$$\Delta T_m = T_m^{\langle \bar{s}s \rangle} - T_m^{\langle \bar{q}q \rangle} = 100^{+36}_{-29}\,\binom{120}{80}\,\mathrm{MeV},$$

The IAM results show that higher energy effects do not affect very much our conclusions and that the ChPT extrapolation from low temperatures seems robust.

To summarize, we have studied the SU(2) and SU(3) temperature evolution of the chiral condensates *in the hadronic phase.* The thermodynamics of the meson gas has been obtained from the virial expansion and Chiral Perturbation Theory. Our results clearly show a significant decrease, about

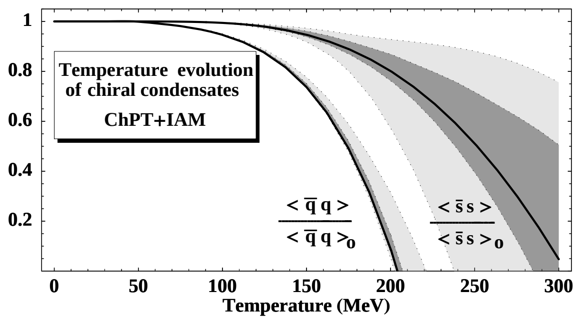

Figure 2 Temperature evolution of chiral condensates using the unitarized ChPT amplitudes. The shaded areas cover the uncertainties in different sets of chiral parameters. Although *they should not really vanish* they are extrapolated down to zero only to ease their comparison and for reference.

20-30 MeV, of the non-strange condensate melting temperature, from the SU(2) to the SU(3) case, similar to lattice results. Of these, about 6 MeV had already been explained with free kaons and etas, but the rest are mainly due to πK and $\pi\eta$ interactions. We have also estimated the effect of heavier hadrons and of the third order virial coefficient. All in all we find

$$T_m^{\langle\bar{q}q\rangle} = 201^{+23}_{-11}\,\text{MeV},$$

In addition, our results show an slower temperature evolution of the strange condensate, shifted by about 80 MeV with respect to the non-strange, due to the different quark masses. More quantitatively, $\langle\bar{q}q\rangle$ does not show a sizable melting up to $T \simeq 150\,\text{MeV}$ and it still remains about 80% when the non-strange is basically zero.

Both effects are clearly visible already at low temperatures. However, we have checked that their size is completely similar when using unitarized models.

These techniques should be easily extended to Heavy Baryon Chiral Perturbation Theory, in order to study the condensates with non-zero baryon density. In general the whole approach could be used with any effective Lagrangian formalism, in particular to study other QCD phase transitions like those of the color superconducting phases.

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